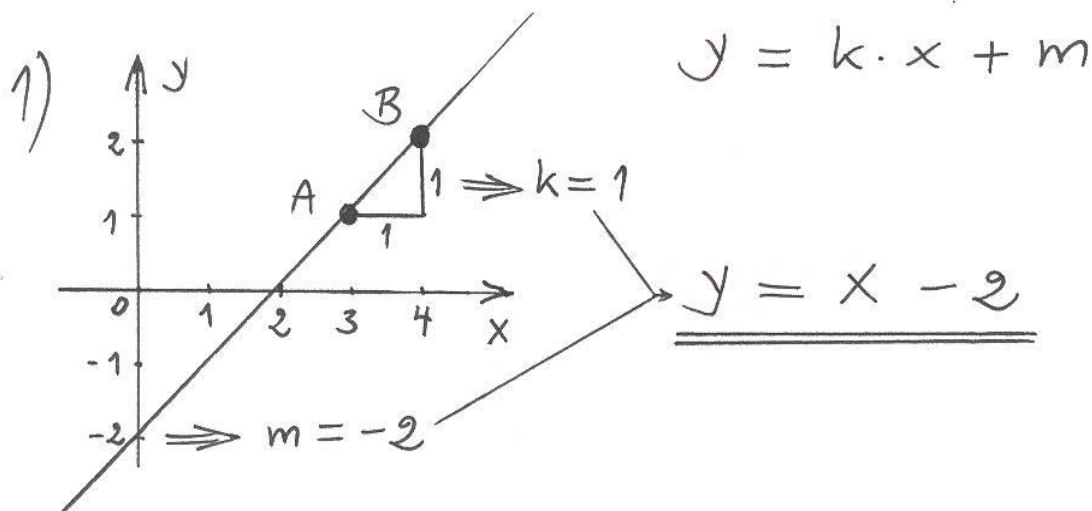


Lösningar till NP Matte 2c, vt 2014

Del B: Utan miniräknare



(2/0/0)

2) a) $11^x = 3 \quad | \lg(\cdot)$

$\lg(11^x) = \lg 3 \quad \text{3:e log-lagen på VL}$

$x \cdot \lg 11 = \lg 3 \quad | / \lg 11$

$x = \frac{\lg 3}{\lg 11}$

(1/0/0)

b) $\lg x = 5 \quad | 10^{\cdot}$

$10^{\lg x} = 10^5$

tar ut värdet

$x = 10^5$

(1/0/0)

3) 12% innebär förändringsfaktorn 1,12.
ökning

(1/0/0)

Alternativ E. $2000 \cdot 1,12^x = 4000$

(dvs $1,12^x = 2$)

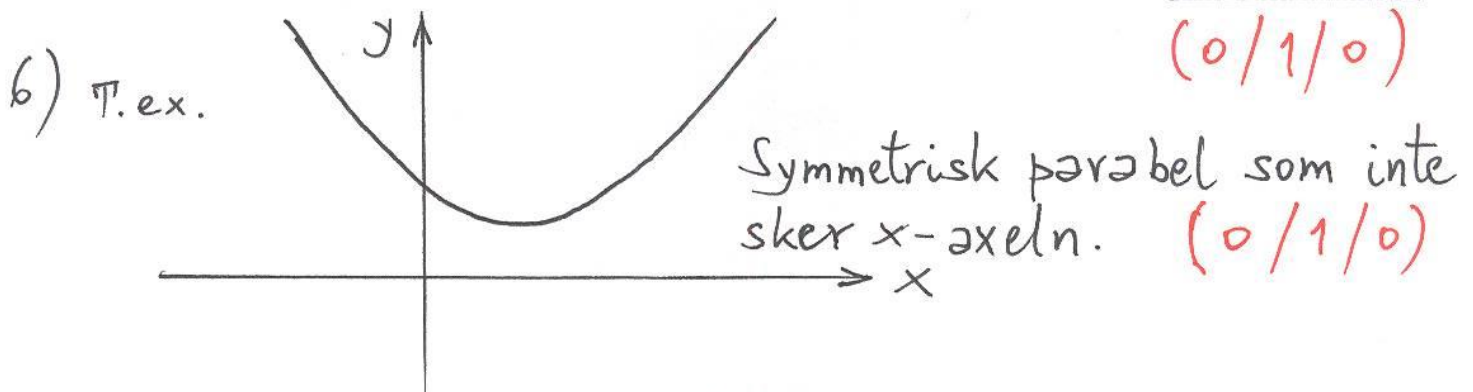
$$4) a) \text{Variationsbredden} = 5,9 - 4,1 = \frac{1,8 \text{ g/cm}^3}{(1/0/0)}$$

$$b) \text{Medianen} = \underline{\underline{5,5 \text{ g/cm}^3}} \quad (1/0/0)$$

c) Standardavvikelsen blir mindre. $(0/1/0)$

$$5) a) (x+5)^2 - (5+x) \cdot (x+5) = (x+5)^2 - (x+5) \cdot (x+5) = \\ = (x+5)^2 - (x+5)^2 = \underline{\underline{0}} \quad (0/1/0)$$

$$b) (3\sqrt{x} - \sqrt{12}) \cdot (3\sqrt{x} + \sqrt{12}) - 7x = \left. \begin{array}{l} \text{Konjugat-} \\ \text{regeln} \end{array} \right\} \\ = 9x - 12 - 7x = 9x - 7x - 12 = \underline{\underline{2x - 12}} \quad (0/1/0)$$



$$7) \text{T.ex. } 3 + 1 = 4 \Rightarrow \boxed{x + y = 4} \\ 2 \cdot 3 - 5 \cdot 1 = 1 \Rightarrow \boxed{2x - 5y = 1} \quad (0/1/0)$$

$$8) (a+4)^2 + (a-4)^2 = a^2 + 8a + 16 + a^2 - 8a + 16 =$$

$$a) = 2a^2 + 32$$

Svar: $\sqrt{2a^2 + 32}$

$(0/1/0)$

$$\begin{aligned}
 8) \text{ b) } \sqrt{2a^2 + 32} &> 10 && | (\cdot)^2 \\
 2a^2 + 32 &> 100 && | -32 \\
 2 \cdot a^2 &> 68 && | /2 \\
 a^2 &> 34 && | \sqrt{\cdot} \\
 \underline{\underline{a > \sqrt{34}}} &&& (0/1/0)
 \end{aligned}$$

$$\begin{aligned}
 9) \lg a^2 + \lg b^2 &= \{3:e \text{ log-l\u00f6sen}\} = 2 \cdot \lg a + 2 \cdot \lg b = \\
 &= 2 \cdot (\lg a + \lg b) = \{1:a \text{ log-l\u00f6sen}\} = 2 \cdot \lg(a \cdot b) = \\
 &= 2 \cdot 5 = \\
 &= \underline{\underline{10}} \\
 & \quad (0/0/1)
 \end{aligned}$$

\u00c5 andra sidan:
 $a \cdot b = 10^5 \quad | \lg(\cdot)$
 $\lg(a \cdot b) = \lg(10^5) = 5$

$$\begin{aligned}
 10) \quad x - \sqrt{3} &= t \\
 x_1 - \sqrt{3} &= 3 && x_2 - \sqrt{3} = 1 \\
 \underline{\underline{x_1 = 3 + \sqrt{3}}} & \quad (0/0/1) && \underline{\underline{x_2 = 1 + \sqrt{3}}}
 \end{aligned}$$

11)

A. $a \cdot a + b \cdot b \neq 0$
 B. $a \cdot a - b \cdot b \neq 0$
 C. $a \cdot b + b \cdot a \neq 0$
 D. $a \cdot b - b \cdot a = 0$

Endast alternativ D uppfyller ekvationen f\u00f6r $\begin{cases} x=a \\ y=b \end{cases}$.

Svar: Alternativ D. $ay - bx = 0$ (0/0/1)

Del C: Utan miniräkneare

$$12) \quad x + 2y = 13 \quad \leftarrow 1 \cdot 2$$

$$2x + 3y = 21$$

$$(I) \quad 2x + 4y = 26$$

$$(II) \quad 2x + 3y = 21$$

$$(I) - (II) \quad 0 + y = 5$$

$$\underline{\underline{y = 5}}$$

$$\rightarrow x + 2 \cdot 5 = 13$$

$$x + 10 = 13 \quad | -10$$

$$\underline{\underline{x = 3}}$$

3:e ekvationen: $2z + x + y = 26$

$$2z + 3 + 5 = 26$$

$$2z + 8 = 26$$

$$2z = 18$$

$$\underline{\underline{z = 9}}$$

(2/0/0)

$$13) \quad x^2 + 2x - 15 = 0$$

$$\left. \begin{array}{l} p=2 \\ q=-15 \end{array} \right\} \Rightarrow x_{1,2} = -1 \pm \sqrt{1+15} = -1 \pm \sqrt{16}$$

$$x_{1,2} = -1 \pm 4$$

a)

$$x_1 = -1 + 4 = 3$$

$$x_2 = -1 - 4 = -5$$

$$\underline{\underline{x_1 = 3}}$$

$$\underline{\underline{x_2 = -5}}$$

(2/0/0)

$$b) \quad x \cdot (x+3) = x+3 \quad \left| \begin{array}{l} / (x+3) \\ \text{om } x \neq -3 \end{array} \right.$$

$$\underline{\underline{x_1 = 1}}$$

(0/2/0)

$$\text{Om } x = -3 : \left. \begin{array}{l} x \cdot (x+3) = x+3 \\ -3 \cdot (-3+3) = -3+3 \\ 0 = 0 \end{array} \right\} \Rightarrow \underline{\underline{x_2 = -3}}$$

$$14) \quad y = -2x + 8,15$$

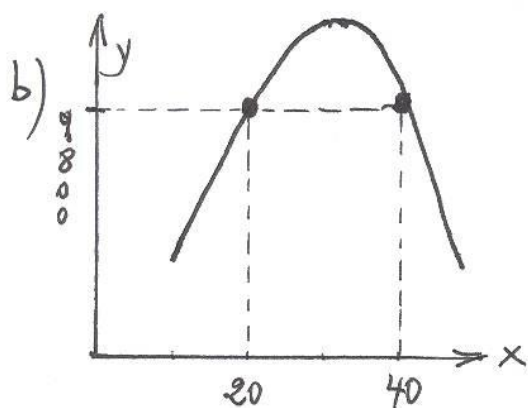
P:s y-koordinat:

$$x=3 \Rightarrow y = -2 \cdot 3 + 8,15 = -6 + 8,15 = 2,15$$

$$A = 3 \cdot 2,15 = \underline{\underline{6,45 \text{ a.e.}}} \quad (2/0/0)$$

$$15) a) \quad y = -6x^2 + 360x + 5000$$

$$x=10 \Rightarrow y = -6 \cdot 100 + 360 \cdot 10 + 5000 = -600 + 3600 + 5000 = 8000 \text{ mg}$$



Symmetri



Även vingmättet 40 mm motsvarar vikten 9800 mg. (1/0/0)

$$\underline{\underline{8000 \text{ mg}}} \quad (1/0/0)$$

$$16) \quad \left. \begin{array}{l} y = 2x + a \\ y = 3x \end{array} \right\} \quad \begin{array}{l} 3x = 2x + a \quad | -2x \\ x = a \end{array}$$

$$2y - x = b$$

$$2y = b + x$$

$$y = \frac{b+x}{2}$$

$$y = 3x$$

$$3x = \frac{b+x}{2} \quad | \cdot 2$$

$$6x = b + x \quad | -x$$

$$5x = b \quad | /5$$

$$x = \frac{b}{5}$$

$$a = \frac{b}{5}$$

eller

$$\underline{\underline{5a = b}}$$

$$(0/2/0)$$

$$17) \quad ax^2 - a^2x = -2 \quad | +2 \quad (a > 0)$$

$$ax^2 - a^2x + 2 = 0 \quad | /a$$

$$x^2 - ax + \frac{2}{a} = 0$$

$$\left. \begin{array}{l} p = -a \\ q = \frac{2}{a} \end{array} \right\} \quad x_{1,2} = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - \frac{2}{a}} \quad : \text{Lösningen}$$

Two different real roots imply: $\frac{a^2}{4} - \frac{2}{a} > 0 \quad | \cdot 4a \quad (a > 0)$

$$a^3 - 8 > 0$$

$$a^3 > 8 \quad | \sqrt[3]{}$$

Svar:

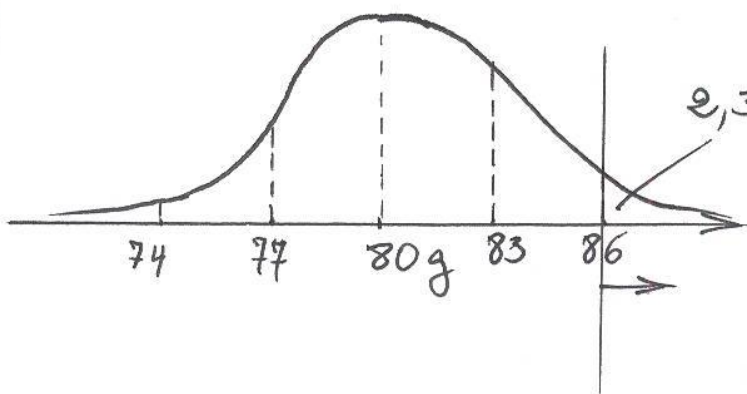
För alla värden $a > 2$

har ekvationen två olika reella rötter.

$$\underline{\underline{a > 2}} \quad (0/0/3)$$

Del D: Med miniräknare

18)

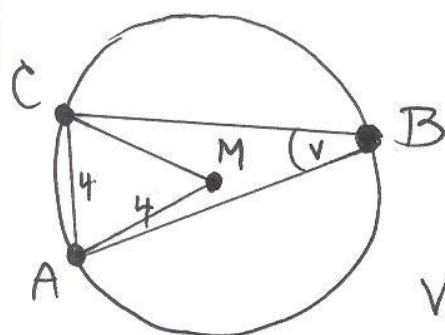


2,3% förväntas väga mer än 86g

$$400 \cdot 0,023 = 9$$

$$\underline{\underline{(2/0/0)}}$$

19)



$MC = 4$ pga radie i cirkeln.

ΔAMC är liksidig

Vinkeln $AMC = 60^\circ =$ Medelpunktsvinkel

Röndvinkeln $v = \underline{\underline{30^\circ}} \quad (2/0/0)$

$$20) a) f(x) = ax^2 + bx + c$$

$$\left. \begin{array}{l} f(0) = a \cdot 0^2 + b \cdot 0 + c = c \\ \text{I grafen avläses } f(0) = 8 \end{array} \right\} \Rightarrow \underline{\underline{c = 8}} \\ (1/0/0)$$

b) I grafen avläses:

Symmetrilinjen $x = 3$

$$\left. \begin{array}{l} |3 - (-5)| = 8 \\ |3 - 10| = 7 \end{array} \right\} \Rightarrow f(10) \text{ närmast} \\ \text{symmetrilinjen} \Rightarrow \underline{\underline{f(10) < f(-5)}} \\ (1/1/0)$$

21) $y =$ Aktiviteten i MBq
 $x =$ Tiden i timmar
 $FF =$ Förändringsfaktorn

Modellen:

$$y = y_0 \cdot (FF)^x$$

↓
Aktiviteten i början

$$a) 4,6 = 11,5 \cdot (FF)^8 \quad | / 11,5$$

$$\frac{4,6}{11,5} = (FF)^8 \quad | (\cdot)^{1/8}$$

$$\left(\frac{4,6}{11,5}\right)^{1/8} = FF$$

$$0,89178 = FF$$

$$\text{Procent} = 1 - FF =$$

$$= 10,8\%$$

$$\underline{\underline{(0/2/0)}}$$

$$b) y = 11,5 \cdot 0,89178^{24} =$$

$$= \underline{\underline{0,74 \text{ MBq}}}$$

$$(0/1/0)$$

22) a) $x =$ Priset på en grå platta
 $y =$ ——— // — svart — //

Uteplats B: $6x + 14y = 1000$ ← $| \cdot 2$

Uteplats A: $12x + 18y = 1422$

(I) $12x + 28y = 2000$

(II) $12x + 18y = 1422$

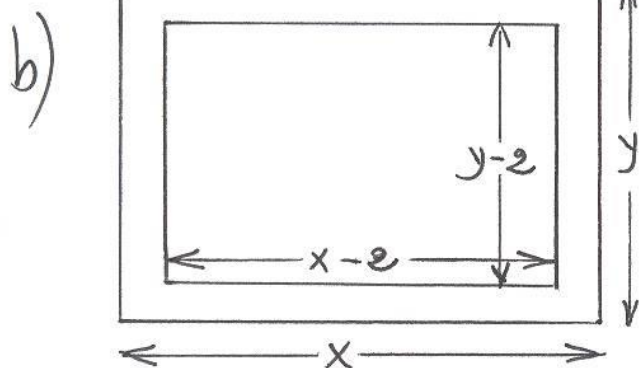
(I) - (II) $0 + 10y = 578$

$10y = 578$

$y = 57,80 \text{ kr}$

(0/3/0)

| /10



$6x + 14 \cdot 57,80 = 1000$

$6x + 809,20 = 1000$

$6x = 190,80$

$x = 31,80 \text{ kr}$

OBS! Andra beteckningar än i a)

Antalet svarta plattor $= 2x + 2y - 4$

— // — grå — // — $= (x-2) \cdot (y-2)$

$K = (2x + 2y - 4) \cdot 57,80 + (x-2) \cdot (y-2) \cdot 31,80 =$

$= 115,60x + 115,60y - 231,20 + (xy - 2x - 2y + 4) \cdot 31,80 =$

$= 115,60x + 115,60y - 231,20 + 31,80xy - 63,60x - 63,60y +$

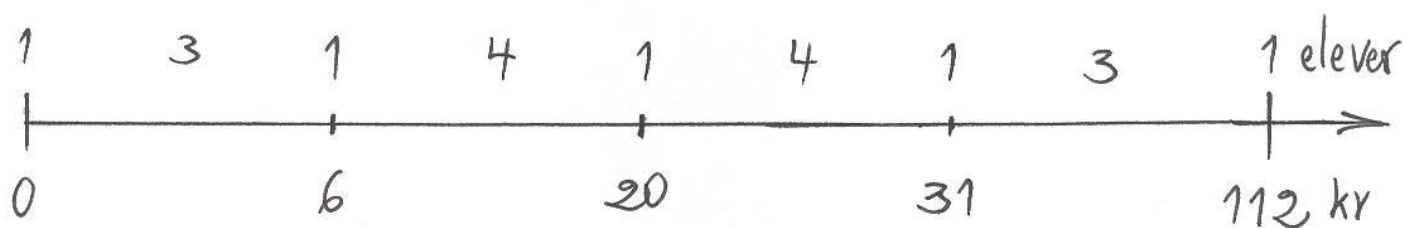
$= \underline{\underline{52x + 52y + 31,80xy - 104}} \quad \left. \begin{matrix} + 127,20 = \\ \end{matrix} \right\}$

(0/0/2)

23) Median = 20 \Rightarrow 9 av de 19 elever är till höger
 9 ————— // ————— vänster
 om medianen 20.

Nedre kvartilen \Rightarrow 4 elever finns till höger
 = 6 4 ————— // ————— vänster
 om nedre kvartilen.

Övre kvartilen \Rightarrow 4 ————— // —————
 = 31 om övre kvartilen.



$$M_{\min} = \frac{4 \cdot 0 + 5 \cdot 6 + 5 \cdot 20 + 4 \cdot 31 + 1 \cdot 112}{19} = 19,3$$

$$M_{\max} = \frac{1 \cdot 0 + 4 \cdot 6 + 5 \cdot 20 + 5 \cdot 31 + 4 \cdot 112}{19} = 38,3$$

\Downarrow

$$19,3 \leq M \leq 38,3$$

Detta utesluter alternativ A. Svar:

Medelvärdet M kan ligga i intervallen B, C och D.

(0/2/1)

24)

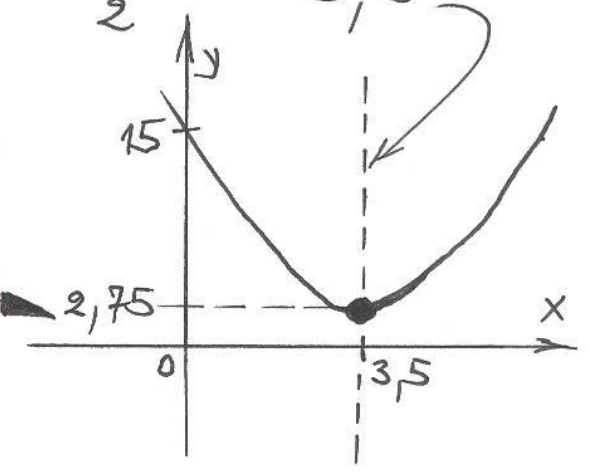
a) $A(x) = g(x) - f(x) = -2x + 15 - (-x^2 + 5x) =$
 $= -2x + 15 + x^2 - 5x = \underline{\underline{x^2 - 7x + 15}}$

(0/0/1)

b) $A(x) = x^2 - 7x + 15$ är en parabel (öppen uppåt).

Symmetrilinjen: $x = -\frac{(-7)}{2} = 3,5$

$A(3,5) = 3,5^2 - 7 \cdot 3,5 + 15 =$
 $= 2,75$

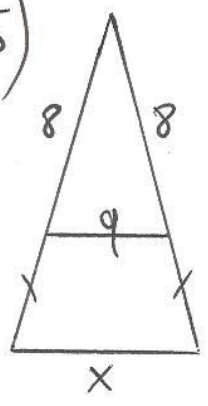


Svar:

Kurvornas minsta avstånd i y-led är 2,75 l.e.

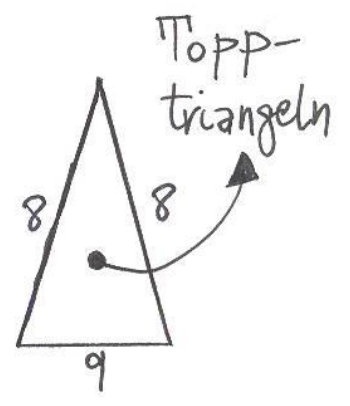
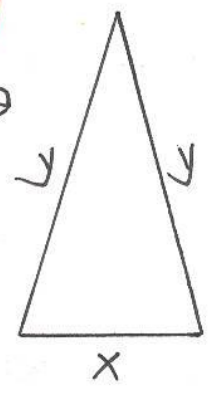
(0/0/2)

25)



Trö likformiga trianglar:

$\frac{x}{9} = \frac{y}{8}$
 $8x = 9y$



Omkretsar:

$x = \left(\frac{9}{8}y\right)$

$x + 2 \cdot (y - 8) + 9 = 9 + 2 \cdot 8$

$x + 2y - 16 = 16$

$x + 2y = 32$

$\frac{9}{8}y + 2y = 32$

$y \left(\frac{9}{8} + 2\right) = 32$

$y \left(\frac{9}{8} + \frac{16}{8}\right) = 32$

$y \cdot \frac{25}{8} = 32$ (0/0/4)

$y = \frac{32 \cdot 8}{25} = 10,24$

$x = \frac{9}{8}y = \frac{9}{8} \cdot 10,24 = 11,52$

Svar: 11,5 cm; 2,2 cm; 2,2 cm; 9 cm.